Individual Round

GLMT 2025

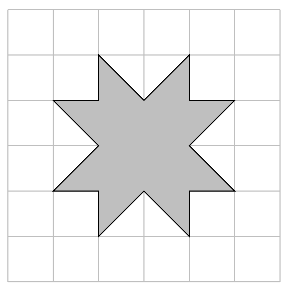
April 19, 2025

1. **[6]** The grid below is filled with the numbers 1, 2, 3, 4, 5, and 6, such that any two consecutive numbers are in adjacent cells. Find the number in the shaded cell.

	1	
3		

2. **[6]** Find
$$\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{25}}}}}}.$$

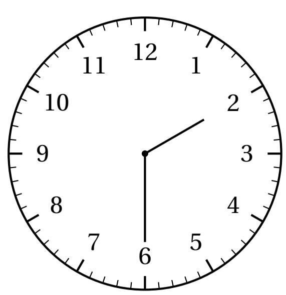
3. **[6]** Ada colors a small star on graph paper (depicted below), where each small square has side length 1 unit. Find the area of the star.



4. **[6]** Find the median of the values
$$\frac{5}{7}, \frac{11}{13}, \frac{3}{5}, \frac{15}{17}, \frac{13}{15}.$$

5. **[6]** Suppose A and B are digits such that $\overline{AB}+\overline{BA}=99$. Find the maximum possible product of A and B.

6. **[7]** A clock is very slow, so it only travels 2 minutes every hour. If the clock is initially (correctly) set to 12:00 pm, find the time it shows when the time should be 2:30 pm on the same day.



7. **[7]** Find the last two digits of $20^{25}+25^{20}$.

8. **[7]** Find the number of real solutions x to the equation
$$x^2 = |2x|.$$

9. **[7]** Find

$$(1)\left(2+\frac{1}{1}\right)\left(3+\frac{1}{2+\frac{1}{1}}\right)\left(4+\frac{1}{3+\frac{1}{2+\frac{1}{1}}}\right)\left(5+\frac{1}{4+\frac{1}{3+\frac{1}{2+\frac{1}{1}}}}\right)\left(6+\frac{1}{5+\frac{1}{4+\frac{1}{3+\frac{1}{2+\frac{1}{1}}}}}\right).$$

10. **[7]** Right triangle ABC has $AB = 6$, $BC = 8$, and $\angle B = 90°$. A circle ω_1 of radius 3 is centered at A, and a circle ω_2 of radius 4 is centered at C. Find the largest possible distance between a point on ω_1 and a point on ω_2.

11. **[8]** In the ground, there are 100 *Digletts* of heights 1 to 100. Three Digletts can form a *Dugtrio* if their heights form an increasing arithmetic sequence. A Diglett can be in multiple Dugtrios. Find the maximum number of distinct Dugtrios that can be formed with the 100 Digletts.

12. **[8]** A positive integer n has three digits in base nine and four digits in base four. Find the number of possible values of n.

13. **[8]** Benicio puts the numbers 1 through 18 in the cells of a 3×6 grid. For each of the ten 2×2 grids within the 3×6 grid they write down the largest entry. Find the largest possible sum of the ten numbers Benicio writes.

14. **[8]** Find the number of real solution pairs (x, y) that satisfy the following equations:

$$x^3 - x^2 = y^3 - y^2,$$
$$x^2 + y^2 = 1.$$

15. **[8]** Bristopher Chrunner is writing a series of 2s and 3s on a blackboard. At one point, the product of all the numbers on the board is 162. When Bristopher Chrunner finishes writing, the sum of all the numbers on the board is 24. Find the number of ordered sequences of numbers that Bristopher Chrunner could have written on the board.

16. **[9]** Find the sum of the roots of the equation

$$x^{25} + \left(\frac{1}{20} - x\right)^{25} = 0.$$

17. **[9]** Two cubes have square bases $ABCD$ and $AEFG$ such that F, A, and C are collinear in that order and D and E lie on the same side of line segment FC. The difference in volumes of the two cubes is 240 and the difference in heights is 5. Find the area of hexagon $BCDEFG$.

18. **[9]** An ice cream cone with radius 2 and height $3\sqrt{2}$ has a hemisphere of radius 2 on top of it such that their bases coincide. Find the side length of the largest cube inscribed in the cone and the hemisphere such that every face of the cube is either parallel or perpendicular to the base of the cone.

19. **[9]** Alicia puts the letters of the word "TRIANGLES" into the cells of a 3×3 grid. Find the number of ways she can arrange the letters so that any two adjacent letters which are adjacent in the word TRIANGLES are in adjacent cells of the grid.

20. **[9]** Let $ABCDE$ be a regular pentagon of side length 1. Let $A'B'CD'E$ be the reflection of $ABCDE$ over CE. If the length of AB' is x, find x^2.